Remarks on models with singlet neutrino in large extra dimensions [1]

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Abstract

Small Dirac masses for neutrinos are natural in models with singlet fermions in large extra dimensions with quantum gravity scale $M_* \sim 1 - 100$ TeV. We study two modifications of the minimal model in order to obtain the mass scale relevant for atmospheric neutrino oscillations with at most $O(1)$ higher-dimensional Yukawa couplings and with $M_* \sim$ a few TeV. 1) In models with singlet fermions in smaller number of extra dimensions than gravity, we find that the effects on $\mathrm{BR}(\mu \to e\gamma)$ and on charged-current universality in $\pi^- \to e\bar{\nu}$, $\mu\bar{\nu}$ decays are suppressed as compared to that in the minimal model with neutrino and gravity in the same space. 2) If small Dirac masses for the singlets are added along with lepton number violating couplings, then the mass scales and mixing angles for neutrino oscillations can be different from those relevant for $\mu \to e\gamma$ and $\pi^- \to e\bar{\nu}$, $\mu\bar{\nu}$. Thus, in both modified models the constraints on M_* from $\mathrm{BR}(\mu \to e\gamma)$ and $\pi^- \to e\bar{\nu}$, $\mu\bar{\nu}$ decays can be significantly relaxed. Furthermore, constraints from supernova 1987a strongly disfavor oscillations of active neutrinos to sterile neutrinos in both the minimal and the modified models.

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1) Introduction

In models with large extra dimensions and low ($\sim$ TeV) quantum gravity scale [1], it is well known that neutrino masses can be naturally small if the singlet (right-handed) neutrino propagates in extra dimensions [2, 3, 4]. In the simplest version of these models, if the quantum gravity scale is smaller than $\sim$10 TeV (as motivated by the hierarchy problem) and if the higher-dimensional Yukawa couplings are at most $O(1)$, then it is difficult to obtain the neutrino mass scale required for a solution to the atmospheric neutrino anomaly. On the other hand, charged-current universality in $\pi^- \to e\bar{\nu}$, $\mu\bar{\nu}$ decays and the limit on $\mathrm{BR}(\mu \to e\gamma)$ constrain the quantum gravity scale to be larger than $10 - 100$ TeV, almost independent of the number of extra dimensions (assuming large $\nu_e - \nu_\mu$ mixing).

In this paper, we discuss two modifications of this model where it is possible to obtain the mass scale required for atmospheric neutrino oscillations even with a quantum gravity scale of a few TeV and Yukawa couplings of $O(1)$. Furthermore, we will show that in these models, this quantum gravity scale can be consistent with charged-current universality in $\pi^- \to e\bar{\nu}$, $\mu\bar{\nu}$ decays and the limit on $\mathrm{BR}(\mu \to e\gamma)$.

Consider for simplicity standard model (SM) gauge singlets $N_I = (\psi_I, \bar{\chi}_I)$ in $5D$, where ψ_I, χ_I are 2-component Weyl spinors. Assume the following terms in the $5D$ action:

$$
\begin{aligned}
S_{\text{free}} &= \int d^4x dy \, i\bar{N}_I \Gamma_a \partial_a N_I + \int d^4x dy \left(\mu_I \bar{N}_I N_I + \text{h.c.} \right) + \int d^4x \, i\bar{\nu}_f \sigma_\mu \partial_\mu \nu_f, \\
S_{\text{int}} &= \int d^4x \left(\frac{\lambda_{fI}}{\sqrt{M_*}} \nu_f \psi_I(y=0) \, h + \frac{\lambda_{fI}^c}{\sqrt{M_*}} \nu_f \chi_I(y=0) \, h + \text{h.c.} \right),
\end{aligned}
$$

(1)

where y denotes the extra dimension assumed to be compactified on a circle of radius R and M_* is the $5D$ "fundamental" Planck scale. λ, λ^c are dimensionless Yukawa couplings in $5D$. We have allowed Dirac mass terms (μ) for singlets and lepton number violating Yukawa couplings, λ^c (assigning lepton numbers $+1$ and -1 to ν_f and N_I, respectively). The index μ runs over $4D$

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while a runs over $5D$ and Γ_a are the "gamma" matrices in $5D$. f is flavor index for SM neutrinos and I denotes the index for the singlets and h is the Higgs doublet. We have chosen the basis for N_I in which the mass matrix μ is diagonal.

In the effective $4D$ theory, ψ_I and χ_I appear as towers of Kaluza-Klein (KK) states, $\psi_I(x,y) = \sum_n 1/\sqrt{2\pi R}\, e^{iny/R} \psi_I^{(n)}(x)$ and similarly for $\chi_I(x,y)$ giving

$$
\begin{aligned}
S_{\text{mass, int}} \;=\; & \int d^4 x \sum_n \left[\left(\mu_I + \frac{in}{R} \right) \psi_I^{(n)} \chi_I^{(n)} + \right. \\
& \left. \frac{\lambda_{fI}}{\sqrt{2\pi}} \frac{M_*}{M_{Pl}} \nu_f \psi_I^{(n)} h + \frac{\lambda_{fI}^c}{\sqrt{2\pi}} \frac{M_*}{M_{Pl}} \nu_f \chi_I^{(n)} h + \text{h.c.} \right].
\end{aligned}
\tag{2}
$$

Here we have used the relation, $M_*^{\delta+2} R^\delta \sim M_{Pl}^2$ (with $M_{Pl} \sim 2.4 \times 10^{18}$ GeV), where δ is the number of extra spatial dimensions all of which are assumed to be of the same size [4]. The KK mass term, $in/R\psi_I^{(n)}\chi_I^{(n)}$, comes from the $5D$ kinetic term. Thus, we see that the $4D$ neutrino Yukawa couplings are suppressed by the volume of the extra dimensions or, in other words, by the ratio M_*/M_{Pl}. When h acquires a vev, we get Dirac mass terms for ν_f with $\psi_I^{(n)}$ and $\chi_I^{(n)}$, denoted by $m_{fI} \sim \lambda_{fI} v M_*/M_{Pl}$ and $m_{fI}^c \sim \lambda_{fI}^c v M_*/M_{Pl}$, respectively ($v \approx 250$ GeV is the Higgs vev). This analysis goes through for any number of extra dimensions.

Consider the "minimal" model with $\mu_I = 0$ and $\lambda_{fI}^c = 0$ and, to begin with, assume one SM neutrino and one N. If $mR \ll 1$, then, to a good approximation, ν, $\psi^{(0)}$ and $\chi^{(n)}$, $\psi^{(n)}$ ($n \neq 0$) form Dirac fermions with masses m and n/R respectively, with mixing between $\chi^{(n)}$ and ν given by $\sim mR/n \ll 1$. $\chi^{(0)}$ decouples and is massless.

For the case of three SM neutrinos ν_f, we can introduce 3 singlets, N_I [5] so that the neutrino Dirac mass matrix is (up to small corrections from

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[4]Of course, $\delta = 1$ is not realistic since in that case R is too large.

[5]The number of 2-component Weyl spinors in a Dirac spinor increases with the number of dimensions. For example, in $6D$, it is 4. Thus, in larger number of extra dimensions, it might be possible to give all 3 SM neutrinos Dirac masses using only 1 or 2 singlets.

ν_f-$\chi_I^{(n)}$ mixing [2, 3])

$$(m_\nu)_{fI} \sim m_{fI} \sim \lambda_{fI} v \frac{M_*}{M_{Pl}} \sim \lambda_{fI} \left(\frac{M_*}{\text{TeV}} \right) 10^{-4} \text{ eV}. \tag{3}$$

The $\nu - \chi^{(n)}$ mixing can have significant effects on weak decays to $l\bar{\nu}$ as follows. The SM neutrino (weak eigenstate) is dominantly the lightest neutrino (with mass m_ν) with small mixture of heavier neutrinos (with mass $\sim n/R$):

$$\nu \approx \frac{1}{N} \left(\nu_L^{(0)} + \sum_{n \neq 0} \frac{mR}{n} \nu_L^{(n)} \right), \tag{4}$$

where $\nu^{(n)}$ are the mass eigenstates. The "normalization factor" is

$$N^2 \approx 1 + \sum_n \frac{(mR)^2}{n^2} \approx 1 + \frac{m^2}{M_*^2} \frac{M_{Pl}^2}{M_*^2} \approx 1 + \frac{\lambda^2 v^2}{M_*^2}, \tag{5}$$

where we have truncated the KK sum at $n/R \sim M_*$, neglecting an $O(1)$ factor in the summation. For $\delta = 2$, the sum is log-divergent and we get an additional factor of $\sim \ln (M_{Pl}/M_*)$ in the sum. Thus, the decay width to $\nu^{(0)}$ is modified compared to SM since $N^2 \neq 1$. Whereas decays to $\nu^{(n)}$'s ($n \neq 0$) (if kinematically allowed) are suppressed by small mixing ($\sim mR/n$) and have a different phase space.

For example, consider the decays $\pi^- \rightarrow e\bar{\nu}$, $\mu\bar{\nu}$ [11]. In the SM, $\Gamma (\pi^- \rightarrow e\bar{\nu})$ is suppressed by m_e^2 due to chirality flip. In the extra dimensional scenario, the chirality flip can occur on the neutrino instead. In other words, π^- decays into e_L and $\nu_R^{(n)} \sim \psi^{(n)}$, i.e., the heavier KK states, through the $m\nu\psi^{(n)}$ term. The large number of KK states up to $m_\pi - m_e$ enhance the effect, whereas the same effect (relative to the SM) in the case of $\pi^- \rightarrow \mu_L\psi^{(n)}$ is smaller. For $\delta = 2$, this effect on $\Gamma (\pi^- \rightarrow e\bar{\nu})$ gives a lower limit on M_* of $O(1000)$ TeV for $\lambda \sim O(1)$ [11]. Of course, $\lambda \sim O(1)$ with $M_* \sim 1000$ TeV gives $m_{\nu_e} \sim 0.1$ eV, which might be too large. The $\pi^- \rightarrow e_L\psi^{(n)}$ decay width scales as $|\lambda|^2 (m_\pi/M_*)^\delta$ so that for $\delta \geq 3$, this effect is smaller than the effect of N^2 on π^- decays into e_R, μ_R and $\nu_L^{(0)} \sim \nu$. This modifies the ratio of decay

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widths to e, μ since the m's and hence the normalization factors are different for ν_e and ν_μ; the lower limit on M_* is $O(10)$ TeV again for $|\lambda_\mu^2 - \lambda_e^2| \sim O(1)$ [11]. We do get the mass scale $\sim 10^{-5}$ (eV)^2 required for a solution to the solar neutrino anomaly via matter-induced oscillations [15] for $M_* \sim O(10)$ TeV and $\lambda \sim O(1)$.

Coherent conversion of SM neutrinos to singlet neutrinos (due to the above mixing) in a supernova (SN) results in energy loss, reducing its active neutrino flux. Since the SM neutrinos (unlike the singlet neutrino) have weak interactions with the matter in the SN core, these oscillations are enhanced by the MSW effect. These resonant oscillations are possible only if the mass of the sterile neutrino state is not larger than $\sim \sqrt{EV} \sim 10$ keV, where $E \sim 100$ MeV is the neutrino energy in a SN and $V \sim 10$ eV is the potential in its core [5]. The survival probability of SM neutrino can be approximated by the product of survival probabilities in each resonance "crossed" by the SM neutrino as it travels out of the SN core, $P_{\nu\nu} \approx \prod_n P_n$ [6, 5]. P_n is approximately independent of the mass of the nth resonance and is given by [6, 5]

$$P_n \approx \exp\left(-\frac{\pi}{2}\frac{4m^2 r_{\text{core}}}{E}\right) \sim \exp\left(-\frac{m^2}{10^{-3}\ (\text{eV})^2}\right), \qquad (6)$$

where $r_{\text{core}} \sim 10$ km is the radius of the SN core. To explain the atmospheric neutrino anomaly via oscillations, we require m^2 (for ν_μ or ν_τ) $\sim \Delta m^2_{\text{atm}} \sim 10^{-3}$ (eV)^2 [15] so that each P_n is $\sim 1/3$. Thus, even if only one resonance is crossed, the energy loss from a SN due to sterile neutrinos will be comparable to that due to active neutrinos. The agreement of the measured duration and number of neutrino events from SN1987a with the prediction of SN models (taking into account theoretical uncertainties) indicates that a new channel for energy loss from the SN should be less effective than the standard neutrino channel; otherwise the neutrino signal duration will be halved, which is not allowed [16]. Therefore, in this case, the measurement of SN1987a neutrino flux implies that no resonance can be crossed so that the mass of the lightest KK state, which is $1/R$, should be larger than ~ 10 keV [5]. This implies

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that the KK states are much heavier than SM neutrinos and hence both the solar and atmospheric neutrino anomalies have to be explained by oscillations among active neutrinos, governed by the mass matrix m_ν above.

Loop diagrams involving KK neutrino tower and longitudinal W contribute to $\mu \to e\gamma$ [7, 8] and $(g-2)_\mu$ [9, 10]. The coefficient of the dimension-5 operator relevant for these two processes, $F^{\mu\nu}\bar{l}\sigma_{\mu\nu}l$ $(l = e, \mu, \tau)$, generated by the KK neutrino exchange, is approximately

$$e\,\frac{m_l}{8\pi^2 v^2}mm^\dagger \sum_n \frac{R^2}{n^2} \sim e\,\frac{m_l}{8\pi^2 v^2}\frac{mm^\dagger}{M_*^2}\frac{M_{Pl}^2}{M_*^2}. \tag{7}$$

The amplitude is enhanced as compared to $4D$ case by the large number of KK states (we have truncated the KK sum at M_*). Since the neutrino masses are given by m (see Eq. (3)), it is obvious that (as in $4D$) there is a direct correlation between neutrino oscillations and the contributions to $\mu \to e\gamma$ from these loop diagrams. For example, in the two flavor case, we get

$$\begin{aligned} \mathcal{A}\left(\mu \to e\gamma\right) &\propto \frac{\left(mm^\dagger\right)_{e\mu}}{M_*^2}\frac{M_{Pl}^2}{M_*^2} \\ &\sim \frac{|m_{\nu_1}^2 - m_{\nu_2}^2|}{M_*^2}\sin 2\theta \frac{M_{Pl}^2}{M_*^2}, \end{aligned} \tag{8}$$

where m_{ν_1}, m_{ν_2} and θ are the masses and mixing angle of the two neutrinos obtained from Eq. (3). This was used in [7] to obtain lower limits on M_* of $O(10-100)$ TeV for $\theta \sim \pi/4$ and $|m_{\nu_1}^2 - m_{\nu_2}^2| \sim 10^{-5}$ (eV)^2 (as relevant for large mixing angle solar oscillations). In the case of three flavors with $2-3$ mixing angle ϕ, but no $1-3$ mixing, the above expression is simply multiplied by $\cos \phi$.

In the minimal model, we see from Eq. (3) that to get $m_\nu^2 \sim \Delta m_{\text{atm}}^2$ we require $M_* \gtrsim 100$ TeV if $\lambda \sim O(1)$. Such high values of M_* are disfavored by the motivation to solve the hierarchy problem [1]. Of course, we can choose $\lambda \gg 1$ and obtain $m_\nu^2 \sim \Delta m_{\text{atm}}^2$ for $M_* \sim 10$ TeV [14] [6], but then the $(4+\delta)D$

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[6]For $M_* \lesssim 10$ TeV, it is not possible to obtain Δm_{atm}^2 even for $\lambda \gg 1$ since, due to the normalization factor, there is an upper limit on m_ν, for a given M_* [2, 14].

theory might reach strong coupling. In other words, from the $4D$ point of view, the Yukawa coupling $\sim \lambda M_*/M_{Pl} \sim m_\nu/v$ is very small at tree level (or at low energies) even though $\lambda \gg 1$ in this case. But, since the $4D$ coupling "runs" with power of energy due to the multiplicity of KK states, it might reach its Landau pole near M_*. For this reason, we will consider $\lambda \overset{<}{\sim} O(1)$ throughout this paper.

With the motivation of obtaining the neutrino mass scale Δm^2_{atm} with $M_* \sim$ TeV and $\lambda \sim O(1)$, we now study two modifications of the minimal model. In the first model, the singlet neutrino propagates in a sub-space of the full extra dimensional space where gravity propagates. In the second model, we consider the effect of non-zero Dirac masses for the singlets and of lepton number violating couplings, λ^c. We will also keep an eye on the correlation between neutrino masses and contribution to $\mu \to e\gamma$ and the effect on $\pi^- \to e\bar{\nu}$, $\mu\bar{\nu}$ in these models.

2) Sub-space

First, consider the case where singlet neutrino propagates in $\delta_\nu < \delta$ dimensions [2]. Assuming that all extra dimensions are of size R, in this case, we get the neutrino Dirac mass matrix

$$(m_\nu)_{fI} \sim m_{fI} \sim \frac{\lambda_{fI}}{\sqrt{(RM_*)^{\delta_\nu}}} v \sim \lambda_{fI} v \left(\frac{M_*}{M_{Pl}} \right)^{\delta_\nu/\delta}. \tag{9}$$

Thus, for $\delta_\nu = 5$ and $\delta = 6$ and with $M_* \sim$ TeV, $\lambda \sim O(1)$, we get $m^2 \sim \Delta m^2_{\text{atm}}$ [2]. To obtain the neutrino mass scale required for a solution to the solar neutrino anomaly via matter-induced oscillations, $\Delta m^2_{\text{sol}} \sim 10^{-5}$ (eV)^2, we can choose the corresponding $\lambda \sim O(0.1)$.

In this case, the dimension-5 operator relevant for $\mu \to e\gamma$ has the coefficient

$$e \frac{m_\mu}{8\pi^2 v^2} \frac{\left(mm^\dagger \right)_{e\mu}}{M_*^2} \sum \frac{R^2}{n^2} \sim e \frac{m_\mu}{8\pi^2 v^2} \frac{\left(mm^\dagger \right)_{e\mu}}{M_*^2} \left(\frac{M_{Pl}^2}{M_*^2} \right)^{\delta_\nu/\delta}$$

$$\propto \frac{|m_{\nu_1}^2 - m_{\nu_2}^2|}{M_*^2} \sin 2\theta \left(\frac{M_{Pl}^2}{M_*^2} \right)^{\delta_\nu/\delta}, \tag{10}$$

where m_{ν_1}, m_{ν_2} and θ are the neutrino masses and mixing angle as obtained from Eq. (9). We see that this contribution is suppressed compared to $\delta_\nu = \delta$ (for the *same* values of m and M_*) due to smaller number of KK states. Thus, the lower limits on M_* of $O(10 - 100)$ TeV obtained for the minimal model [7] (for $\theta \sim \pi/4$ and $|m_{\nu_1}^2 - m_{\nu_2}^2| \sim 10^{-5}$ (eV)^2) can be relaxed by a factor of $O(20)$, assuming $\delta_\nu = 5$ and $\delta = 6$. The lower limit now becomes $M_* \sim$ few TeV.

In terms of λ (instead of m), the coefficient of the dimension-5 operator is

$$e \, \frac{m_\mu}{8\pi^2} \frac{\left(\lambda\lambda^\dagger\right)_{e\mu}}{M_*^2}, \tag{11}$$

i.e., for *fixed* λ, it is independent of δ_ν or δ [8]. But, with $\delta_\nu = 5$, $\delta = 6$ and $M_* \sim$ few TeV, we require $\left(\lambda\lambda^\dagger\right)_{e\mu} \sim O(10^{-2})$ to obtain the mass scale for solar neutrino oscillations (as shown above). Whereas with $\delta_\nu = \delta$ and for $M_* \sim O(10 - 100)$ TeV, we require $\left(\lambda\lambda^\dagger\right)_{e\mu} \sim O(1)$. Hence the above coefficient is the same for these two parameter sets (which give the same neutrino masses), in agreement with the analysis in terms of m.

Reference [8] also considers the constraints from $\mu \to 3e$ and $\mu \to e$ conversion in nuclei. For these processes, loop contribution due to KK neutrino tower to the effective Z-μ-e coupling (in addition to the γ-μ-e coupling) has to be included. This coupling depends on $\sim \lambda^4 v^2/M_*^2$ [8] (dropping the flavor indices on λ for simplicity), unlike the γ-μ-e coupling which depends on $\sim \lambda^2 v^2/M_*^2$ as above. The experimental bounds on $\mu \to 3e$ and $\mu \to e$ conversion in nuclei give the lower limit $M_*/\lambda^2 \sim 200 - 300$ TeV [8] which, for $\lambda \sim 1$, is stronger than that from $\mu \to e\gamma$. However, in this sub-space case, as mentioned above, to get $m_\nu^2 \sim \Delta m_{\text{sol}}^2$ with $M_* \sim$ few TeV, we require $\lambda \sim O(0.1)$ and hence this quantum gravity scale is consistent with $\mu \to 3e$ and $\mu \to e$ conversion in nuclei.

In this sub-space scenario, the effect on $\pi^- \to e\bar{\nu}$, $\mu\bar{\nu}$ decays is due to the normalization factor N^2 (since $\delta_\nu = 5$) and thus also depends on $\sum_n (mR)^2/n^2$ (see Eq. (5)). As above, this factor is smaller than in the

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minimal model for the same values of m and M_*. Therefore, the lower limit on M_* from $\pi^- \to e\bar{\nu}$, $\mu\bar{\nu}$ decays is also reduced from $O(10)$ TeV (obtained for the minimal model [11], assuming $\lambda \sim O(1)$ which gives $m_\nu^2 \sim 10^{-5}$ (eV)^2) to $\sim$ few TeV.

With $\delta = 6$, we get $1/R \sim O(10 - 100)$ MeV so that the SN1987a constraint is satisfied.

3) See-saw

Next, consider the case with non-zero Dirac mass $\mu_I(\ll M_*)$ for the singlets [8, 12, 13]. In the limit $\mu \gg m$ and $m^c \approx 0$, the linear combinations $\sim \left(\chi_I^{(n)} + m_{fI}/\sqrt{\mu_I^2 + n^2/R^2}\, \nu_f\right)$ form Dirac pairs with $\psi_I^{(n)}$'s of masses $\sim \sqrt{\mu_I^2 + n^2/R^2}$. Of course, with $\lambda^c \approx 0$, there are 3 massless neutrinos which are dominantly ν_f's, even though a priori all fermions have mass terms. The reason is that in this case we can define a conserved lepton number with charges $+1$ for ν and χ and -1 for ψ so that only Dirac masses are allowed. Thus, there are 3 "unpaired" fermions with charge $+1$ which are the ν_f's. In the minimal model ($\mu = 0$), these massless fermions are $\chi_I^{(0)}$'s as mentioned earlier. We require lepton number violation, for example, $m^c \neq 0$, so that these massless neutrinos can get Majorana masses $\nu_f \nu_{f'}$ [12, 13]. Then, the see-saw mechanism (see Fig. 1) gives

$$(m_\nu)_{ff'} \sim \sum_I \sum_n \frac{m_{fI}\mu_I m_{f'I}^c + m_{fI}^c \mu_I m_{f'I}}{\mu_I^2 + n^2/R^2}. \tag{12}$$

In this case, one singlet N suffices to give masses to all 3 SM neutrinos, but in general, one can have many singlets with different μ_I's.

In the case of $\delta \geq 2$, the see-saw is "divergent" due to $\sum_n 1/n^2$. If we truncate the KK sum at M_*, then we get the neutrino Majorana mass matrix

$$\begin{aligned}
(m_\nu)_{ff'} &\sim \sum_I \left(m_{fI}\mu_I m_{f'I}^c + m_{fI}^c \mu_I m_{f'I}\right) R^\delta M_*^{\delta-2} \\
&\sim \left(m\mu m^{c\,T} + m^c \mu m^T\right)_{ff'} \frac{M_{Pl}^2}{M_*^4} \\
&\sim \left(\lambda\mu\lambda^{c\,T} + \lambda^c \mu \lambda^T\right)_{ff'} \frac{v^2}{M_*^2}.
\end{aligned} \tag{13}$$

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$$\nu_f \qquad m_{fI} \qquad \psi_I^{(n)} \qquad \mu_I \qquad \chi_I^{(n)} \qquad m_{f'I}^c \qquad \nu_{f'}$$
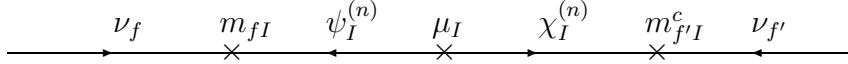
Figure 1: See-saw mechanism for generating SM neutrino masses in the case $\mu_I \neq 0$ and $m^c \neq 0$. There is another diagram obtained by $m \leftrightarrow m^c$, $\psi_I^{(n)} \leftrightarrow \chi_I^{(n)}$.

The KK sum is log-divergent for $\delta = 2$ so that the above result is multiplied by a factor of $\sim \ln\left[M_*^2/\left(\mu^2 + 1/R^2\right)\right] \sim O(10)$ and was mentioned in [12]. We see from Eq. (13) that for $M_* \sim$ TeV, $\delta \geq 2$ and λ, $\lambda^c \sim O(1)$, we get $m_\nu^2 \sim \Delta m_{\text{atm}}^2$ by choosing $\mu \sim$ eV. If all singlets have Dirac masses of $O(\text{eV})$, then to get $m_\nu^2 \sim \Delta m_{\text{sol}}^2$, we can choose the corresponding λ, $\lambda^c \sim O(0.1)$.

We next consider the constraints from SN1987a on this model. The analysis is different than in the minimal model since in this case $m_\nu \not\propto m$ (the latter governs the mixing between SM and sterile neutrinos and hence the energy loss in SN1987a) and also the masses of the sterile neutrinos are given by $\sim \sqrt{\mu^2 + n^2/R^2}$ instead of n/R. Thus, one way to evade the SN1987a constraint, independent of the number and the size of the extra dimension, is to choose $\mu_I \stackrel{>}{\sim} 10$ keV so that no resonance can be crossed in the SN1987a core. Then, the solar and atmospheric neutrino oscillations have to involve only active neutrinos. With $\mu \sim 10$ keV, we require $\lambda\lambda^c \sim O(10^{-4})$, $O(10^{-5})$ to get $m_\nu^2 \sim \Delta m_{\text{atm}}^2$, Δm_{sol}^2, respectively, i.e, λ, $\lambda^c \sim O(1)$ will give too large m_ν (see Eq. (13)).

If $\delta \geq 4$, then we get $1/R > 10$ keV so that again no resonance is crossed and the SN1987a constraint is satisfied for any value of μ.

The remaining cases are $\mu_I \ll 10$ keV and $\delta = 2, 3$. The number of resonances crossed, i.e., the number of sterile neutrino states lighter than 10 keV, is $n_{\text{res}} \sim (R\ 10\ \text{keV})^\delta$, and we have to consider each value of δ

separately.

For $\delta = 2$ and $M_* \sim 1-10$ TeV, we get $1/R \sim 0.01$ eV so that $n_{\mathrm{res}} \sim 10^{12}$. As mentioned earlier, we require $P_{\nu\nu} \approx (P_n)^{n_{\mathrm{res}}} \overset{>}{\sim} 1/2$ so that the SN energy loss due to sterile neutrinos is less than that due to active neutrinos. Thus, we get the constraint m^2, $m^{c\,2} \overset{<}{\sim} 10^{-15}$ (eV)^2. Then, the active neutrino masses are too small to account for solar and atmospheric neutrino oscillations. The $\nu_e - \chi^{(n)}/\psi^{(n)}$ mixing is also too small to be relevant for explaining the solar neutrino anomaly by oscillations of ν_e to sterile neutrinos. Thus, $\mu \ll 10$ keV is ruled out by SN1987a constraint.

For $\delta = 3$ and $M_* \sim 1 - 10$ TeV, we get $1/R \sim O(100 \text{ eV} - 1 \text{ keV})$ so that $n_{\mathrm{res}} \sim 10^3 - 10^6$ [7]. Hence, the SN1987a constraint, $P_{\nu\nu} \sim 1$, requires m^2, $m^{c\,2} \overset{<}{\sim} 10^{-6} - 10^{-9}$ (eV)^2, i.e., λ, $\lambda^c \sim O(1)$ might be allowed (depending on M_*). Thus, we can choose $\mu \sim$ eV to get $m_\nu^2 \sim \Delta m_{\mathrm{atm}}^2$ for $M_* \sim$ TeV and λ, $\lambda^c \sim O(1)$ while (marginally) satisfying the SN1987a constraint.

Of course, for $\delta \geq 3$ and for any value of μ, the sterile neutrinos are heavier than $1/R \overset{>}{\sim} 100$ eV so that they cannot be directly involved in oscillations of SM neutrinos in the sun or in the atmosphere.

The coefficient of the dimension-5 operator, $F^{\mu\nu}\bar{l}\sigma_{\mu\nu}l$, generated by exchange of KK neutrinos, is similar to the earlier case, with an additional contribution from m^c:

$$e\,\frac{m_l}{8\pi^2 v^2}\,\frac{mm^\dagger + m^c m^{c\,\dagger}}{M_*^2}\,\frac{M_{Pl}^2}{M_*^2}. \tag{14}$$

Thus, we see that both $(g-2)_\mu$ and $\mathrm{BR}(\mu \to e\gamma)$ depend on $\left(mm^\dagger + m^c m^{c\,\dagger}\right)$ whereas m_ν depends on $\left(m\,\mu\,m^{c\,T} + m^c\,\mu\,m^T\right)$. Therefore, it is clear that the parameters for neutrino oscillations and for $\mu \to e\gamma$, $(g-2)_\mu$ may not be related in this class of models. In particular

$$\mathcal{A}\left(\mu \to e\gamma\right) \propto \left(mm^\dagger + m^c m^{c\,\dagger}\right)_{e\mu} \not\propto |m_{\nu_1}^2 - m_{\nu_2}^2|\sin 2\theta, \tag{15}$$

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[7]R and hence n_{res} is very sensitive to the value of M_*.

where the masses m_{ν_1}, m_{ν_2} and mixing angle θ (relevant for ν_e-ν_μ oscillations) are obtained from m_ν in Eq. (13).

To illustrate the above point, consider for simplicity only ν_e and ν_μ and suppose we have 2 singlets with the same μ's. Assume

$$m \;\propto\; \begin{pmatrix} 0 & 0 \\ \rho & 1 \end{pmatrix}, \tag{16}$$

$$m^c \;\propto\; \begin{pmatrix} 1 & 0 \\ 0 & \sigma \end{pmatrix} \tag{17}$$

with ρ, $\sigma \sim O(1)$. In this example, $\mu \to e\gamma$ depends on

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 + \sigma^2 + \rho^2 \end{pmatrix}, \tag{18}$$

whereas

$$m_\nu \propto \begin{pmatrix} 0 & \rho \\ \rho & 2\sigma \end{pmatrix} \tag{19}$$

so that $\nu_\mu - \nu_e$ mixing can be large (with non-degenerate neutrinos) as required for solar oscillations with large mixing angle, but the loop contribution to $\mu \to e\gamma$ is zero.

Even if the flavor structures of m and m^c are similar, $\mathrm{BR}(\mu \to e\gamma)$ might not be correlated with neutrino masses since, for given M_*, $\mu \to e\gamma$ depends only on λ and λ^c, whereas m_ν depends also on μ. For $\delta = 2$, as explained above, to satisfy the constraints from SN1987a, we might have to choose $\mu \sim 10$ keV and hence $\lambda\lambda^c \sim 10^{-5}$ to get $m_\nu^2 \sim \Delta m_{\mathrm{sol}}^2$. Then, $M_* \sim$ few TeV is consistent with the limit on $\mathrm{BR}(\mu \to e\gamma)$. For $\delta \geq 3$, μ is not constrained by SN1987a. So, we can choose the relevant λ, λ^c small enough such that $M_* \sim$ few TeV is consistent with $\mu \to e\gamma$ and, at the same time, we can get the required m_{ν_e}, m_{ν_μ} by choosing the corresponding μ_I's appropriately. As mentioned in section 2, loop contributions to $\mu \to 3e$ and $\mu \to e$ conversion in nuclei also depend on λ and λ^c and hence are suppressed if λ, λ^c are small.

A similar analysis shows that $M_* \sim$ few TeV can be consistent with $\pi^- \to e\bar{\nu}$, $\mu\bar{\nu}$ since, for given M_* and for $\mu \ll m_\pi$, the effect on π^- decays also depends only on λ and λ^c.

4) Conclusion

In summary, we have studied two "non-minimal" models with singlet neutrino in large extra dimensions and TeV scale quantum gravity. These models can accommodate the mass scale relevant for atmospheric neutrino oscillations even with $O(1)$ higher-dimensional Yukawa couplings and $M_* \sim$ a few TeV, unlike the minimal model. The first model has singlet neutrino propagating in smaller number of extra dimensions as compared to gravity whereas the second model has small Dirac mass terms for the singlets and lepton number violating couplings. In both models, the constraints on M_* from $\mathrm{BR}(\mu \to e\gamma)$ and $\pi \to e\bar{\nu}$, $\mu\bar{\nu}$ decays can be significantly weakened as compared to the minimal model so that $M_* \sim$ few TeV is consistent with these decays. Also, due to the SN1987a constraint, active-sterile neutrino oscillations are strongly disfavored in both the minimal and the modified models.

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